Heat Transfer and Population Characteristics of Dispersed Evaporating Droplets

The direct-contact heat transfer to a dispersed population of immiscible evaporating droplets is analyzed. Also studied is a population balance formulation for the distribution of two-phase bubbles, which is similar to that utilized in particle and droplet dispersion analysis and is capable of including bubble coalescence and break-up. It is shown that the method of classes is useful particularly in solving such problems when the growth functions are size- and time-dependent and nonquadratic. The method is applied to a liquid cool-down, representing the initial chilling stage of a direct-contact batch crystallizer or cold-storage unit wherein a vessel containing a liquid is chilled by injecting a dispersion of refrigerant droplets. Transient bubble population characteristics, volumetric heat transfer coefficient, total heat transfer, and liquid temperature are predicted, along with the liquid refrigerant holdup.

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Introduction

Heat and mass transfer to dispersed droplets in an immiscible continuous medium are direct-contact transport processes which are utilized in many industrial operations, including spray combustion, spray cooling, water desalination and purification, and solar and geothermal power (Jacobs, 1988). Heat and mass transfer are, in fact, the limiting mechanisms in many such processes, controlling material throughput, product quality, and system response and size. In the study presented here, we are concerned particularly with systems that comprise a dispersion of evaporating liquid droplets in a surrounding immiscible liquid, a direct-contact heat transfer process which has had particular importance in water desalination (Wiegandt, 1958; Spiegler, 1966), crystallization (Byrd and Mulligan, 1986), solar and geothermal power systems (Boehm, et al., 1974; Jacobs, 1988b), and thermal storage (Wright, 1988).

In designing and analyzing such systems, it is especially important to evaluate the rate of change of the dispersed droplets, the rate of evolution of gases, the overall heat transfer characteristics, and the overall coefficients of heat transfer. An analytical method utilizing a statistical mechanical approach is presented here which potentially is capable of quantifying the behavior and characteristics of such a dispersion, while including the effects of changing bubble size spectra, break-up, coalescence, and other physical phenomena which may be present. The method is then applied to a determination of a dispersed-phase evaporative cool-down, which illustrates the transient behavior of a dispersed-phase liquid thermal-storage unit, the unsteady initial stages of dispersed-refrigerant batch crystallization, and the transient periods of various other unit operations.

Definition of Problem

Consider a process vessel containing a primary liquid into which droplets of a secondary liquid are fed. As the droplets enter, with a given size distribution, they begin evaporating and withdrawing heat from the primary liquid, thus beginning a cool-down process. The entering droplets are assumed to be at the saturation temperature corresponding to the pressure within the vessel, which is taken to be uniform and constant as a first-order representation. Also, the primary liquid in the vessel is assumed to be uniform for this work and immiscible with respect to either of the phases of the evaporating, dispersed fluid. As the droplets complete their evaporation, the vapor exits the liquid through the free surface and is subsequently withdrawn from the vessel. In general, the primary liquid can be considered either a fixed quantity contained within the vessel or a fluid circulating through inlet and outlet ports. Internally, it is assumed that the relative motion which always exists in such systems between the evaporating droplets and the continuous liquid can be quantified on average and specified at the outset, whether caused by stirring, buoyancy, or by other means.

Such systems have been studied which utilize various aqueous solutions (including water) as the continuous liquid and hydrocarbons (freon, butane, pentane) as the evaporating, dispersed liquid. A condensing operation would essentially be the reverse of this process, with vapor bubbles injected, liquid condensate removed, and an increase occurring in the temperature of the continuous liquid. Although the study presented here is primarily for evaporative systems, the same procedures and methods are assumed to be applicable to condensing systems as well with only minor modifications.

The mechanisms of evaporation of a single liquid droplet in a confining immiscible liquid are extremely complicated and have been studied for some time. For example, consider a subcooled single liquid droplet which is injected into an otherwise continuous immiscible liquid. If the droplet contains adequate impurities such that vapor nucleation can readily occur, then evaporation should begin when the droplet reaches the saturation temperature corresponding to the surrounding pressure, assuming this temperature is cooler than the surrounding medium. As evaporation progresses, the droplet mass becomes a two-phase entity of liquid and vapor contained within an envelope of the surrounding immiscible liquid. Such a two-phase configuration of a bubble has been referred to as a "drobble" (Sudhoff et al., 1982).

Figure 1 shows this configuration for the case of the surface tension of the droplets greater than that of the surrounding liquid. The geometry of the liquid phase is governed by surface tension, while its temperature or degrees of superheat is controlled by nucleation. The degree of superheating and the mechanisms of bubble nucleation in a single superheated drop suspended in an immiscible liquid has been studied by Moore (1959), while the possibility of boiling in such drops was studied by Iida and Takashima (1980) and examined by Mori and Komotori (1976) using high-speed photography. Experimentally, nucleation has been achieved by the injection of air, nitrogen, and even electrically (Prakash and Pinder, 1967; Tochitani et al., 1977). A nucleating vapor or gas bubble within the liquid droplet may separate from the liquid or remain within the liquid boundary (Moore, 1959), depending on the surface tensions of the dispersed liquid medium and the continuous liquid. Gradon and Selecki (1976, 1977) discuss the importance of the surface tension of both liquids and their effect on the behavior of the evaporation. When the surface tension of the continuous phase is higher than that of the liquid droplet and the average density of the droplet is less than the density of the continuous phase, the growing two-phase bubble will move relative to the surrounding liquid, with vapor accumulating at the top of the envelope. Thus, the droplet mass begins as a liquid droplet and ends as a vapor bubble, and in the interim expands as a two-phase bubble. Mori (1978, 1985) discusses four types of configurations for two-phase bubbles, as determined exclusively by interfacial tension.

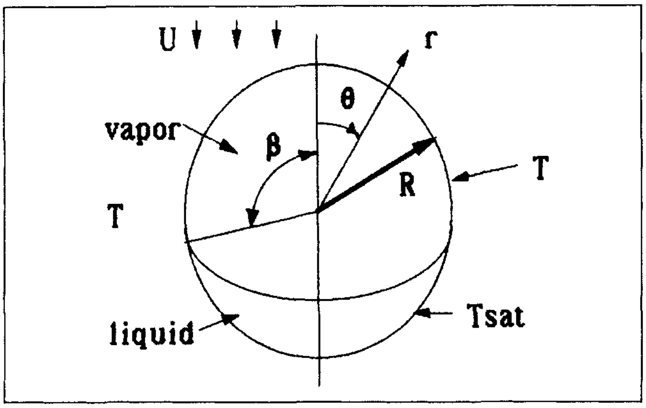

Figure 1. Two-phase bubble configuration.

From photographic evidence (Sideman and Taitel, 1964), when 1% of the mass of the droplet has evaporated, most of the bubble is occupied by vapor. Thus, during most of the evaporation process the motion of the two-phase bubble is much like that of a pure vapor bubble, and influences the overall shape. Depending on the initial size of the liquid droplet, the growing bubble may stay spherical (Tochitani, et al., 1977) or become elliptical or cap-shaped (Sideman and Taitel, 1964; Simpson et al., 1974). Heat transfer, however, strongly depends on both the geometry and the fraction of the surface area of the envelope which is internally wetted and varies during most of the evaporation process. The heat transfer coefficient of evaporating two-phase bubbles has been studied both theoretically and experimentally. Sideman and Isenberg (1967) theorized that heat transfer between the bubbles and the surrounding liquid medium occurs only through the liquid-liquid interface and is a function of the mean bubble diameter. Sideman et al. (1965) describe the dependence on diameter by a power law. Simpson et al. (1974b) report that such a bubble oscillates from side to side, causing the unevaporated liquid to slosh. They assumed that the oscillation causes a liquid film to form on the inside surface of the envelope, thus increasing the liquid-liquid surface area and thus the heat transfer coefficient. Raina and Grover (1985) incorporated this effect into their theoretical model.

The relative motion of a two-phase bubble has also been of interest because of the potential influence of the external convective conductance on the overall heat transfer to the evaporating liquid. For example, as droplets evaporate and buoyantly rise through a continuous liquid, their velocity changes. Sideman and Taitel (1964) report an increase in velocity after 0.1% evaporation. Simpson et al. (1974b) note a nearly constant velocity until the diameter reaches its critical limit and then an increase in velocity. Motion of an evaporating droplet due to buoyancy has been studied theoretically and experimentally by Selecki and Gradon (1977), Tochitani et al. (1977), and Raina et al. (1984). The latter authors have also included this effect in their theoretical heat transfer models.

The purpose of this paper is to present a method of determining the statistical characteristics of an evaporating bubble distribution as well as the overall heat transfer and volumetric heat transfer coefficients. The method is then applied to a determination of the transient cool-down of the continuous-phase temperature for the specific case of a closed (and adiabatic) liquid thermal storage or batch crystallizer system. It is shown that by using this method, the behavior of such systems potentially can be predicted from single-bubble heat transfer correlations, particularly when such correlations are adequately corrected for the bubble interactions which are expected to occur, as suggested by Jacobs (1988), for media of low to moderate holdup.

Population Balance Analysis

It is seen that much has been established to date concerning heat transfer to single evaporating droplets. It must also be acknowledged, however, that at this point such activity is very much an open area of research. In the work presented here, we assume that adequate correlations can be derived from the theoretical and experimental work which has been carried out for single-droplet heat transfer and attempt to incorporate this in an analysis which can predict the behavior of a dispersion of such two-phase bubbles. A population balance approach, which has been widely used in the analysis of particle dispersions (Randolph and Larson, 1988), is used here to consider a size distribution of dispersed two-phase bubbles.

As shown in previous studies, it is possible to classify population balance approaches into two main groups: methods of moments (Byrd and Mulligan, 1986) and methods of orthogonal collocation. The method of moments, as originally proposed by Hulburt and Katz (1964), has been widely used even though difficulties are encountered when growth rates are size-dependent and the local heat transfer coefficient does not have a quadratic form. In the case of bubble dispersions, the heat transfer has been shown to be a complex function of not only the bubble size but motion and bubble interaction (Gal-Or and Walatka, 1967). Thus, to apply the population balance methodology for the prediction of a bubble size distribution, it is necessary to employ another solution method offering more flexibility. Orthogonal collocation methods involve the decomposition of the density function on a set of orthogonal functions, with additional mathematical transformations of the population balance into a set of linear, ordinary differential equations. Marchal et al. (1988) proposed a new method of classes, which is capable of considering size-dependent growth rate and agglomeration so that it can be used in the analysis of bubble distributions.

Although it was developed for crystallization studies (particle systems), the method of classes is modified in this study for use in dispersed phase, evaporative direct-contact heat transfer (bubble systems). The modified method is developed to predict the transient and steady-state bubble population density and overall heat transfer to the distributed bubbles in an immiscible liquid system, including size- and temperature-dependent growth rate, a nonquadratic heat transfer coefficient, and bubble breakage and random coalescence.

A population balance of droplets within a dispersion having a radius between R and $R + dR$ during time dt, with no specification regarding location within the system can be represented as

$$Q_e\Psi_e dRdt + r_A dRV_T dt = Q_s\Psi_s dRdt + d(\Psi G)V_T dt + r_B dRV_T dt + d(\Psi V_T)dR \quad (1)$$

where

Ψ = number-density distribution function = $\Psi(R, t)$
Q = volumetric flow rate of inlet or exit
$V_T(t)$ = total volume of system (changes during evaporation)

and $\Psi(R, t)V_T dR$ is the number of droplets in the size range dR at time t in the volume V_T of the system. The subscript e refers to entering conditions of the dispersed droplets and s refers to the exit conditions. Also,

$G = dR/dt = G(R, t)$ = bubble growth function
r_A = net rate of appearance of bubbles in R to $R + dR$ size range due to coalescence
r_B = net rate of disappearance of bubbles in R to $R + dR$ size range due to breakup

Then, by manipulation, the population balance of the dispersed bubbles can be put in the classical form

$$\frac{1}{V_T}\frac{\delta(\Psi V_T)}{\delta t} + \frac{\delta}{\delta R}(\Psi G) + \frac{Q_s\Psi - Q_e\Psi_e}{V_T} = r_A - r_B \quad (2)$$

To solve this partial differential equation, the method of classes (Marchal et al., 1988) is applied to develop a set of ordinary differential equations. Let $R_0, R_1, \ldots R_N$ be a set of bubble radii with R_0 the smallest radius of a bubble and R_N the largest radius of a bubble. These sizes denote N classes, each noted C_i, where the width of a class C_i is

$$\Delta C_i = R_i - R_{i-1}$$

and the characteristic size of the particles of class C_i is S_i, that is

$$S_i = (R_{i-1} + R_i)/2$$

Let $N_i(t)$ be the number of bubbles in class C_i at time t somewhere in the volume V_T,

$$N_i(t) = \int_{R_{i-1}}^{R_i} \Psi(R, t)\, dR$$

Integrating the population balance Eq. 2 between R_{i-1} and R_i yields

$$\frac{dN_i}{dt} + \frac{1}{V_T}\frac{dV_T}{dt}N_i + [G(R_i)\Psi(R_i) - G(R_{i-1})\Psi(R_{i-1})] \cdot \frac{Q_sN_i - Q_eN_{i,e}}{V_T} = R_{A,i} - R_{B,i} \quad (3)$$

where

$R_{A,i} = \int_{R_{i-1}}^{R_i} r_A\, dR$ = net number of bubbles in volume V_T generated in class C_i after coalescence
$R_{B,i} = \int_{R_{i-1}}^{R_i} r_B\, dR$ = net number of bubbles in volume V_T disappearing from class C_i after breakage
$N_{i,e}$ = number of particles per unit volume of inlet flow

Assuming Ψ remains a constant = ϕ_i on C_i, then

$$\Psi(R_i) = \frac{\phi_{i+1} + \phi_i}{2} = \frac{N_{i+1}}{2\Delta C_{i+1}} + \frac{N_i}{2\Delta C_i}$$

$$N_i(t) = \phi_i(t)\Delta C_i$$

The set of equations then becomes

$$\frac{dN_1}{dt} + \frac{1}{V_T}\frac{dV_T}{dt}N_1 + \frac{Q_sN_1 - Q_eN_{1e}}{V_T} + \frac{G(R_1)}{2\Delta C_2}N_2 + \frac{G(R_1)}{2\Delta C_1}N_1 = R_{A,1} - R_{B,1} \qquad (4a)$$

$$\frac{dN_i}{dt} + \frac{1}{V_T}\frac{dV_T}{dt}N_i + \frac{Q_sN_i - Q_eN_{ie}}{V_T} + \frac{G(R_i)}{2\Delta C_{i+1}}N_{i+1} + \frac{G(R_i) - G(R_{i-1})}{2\Delta C_i}N_i - \frac{G(R_{i-1})}{2\Delta C_{i-1}}N_{i-1} = R_{A,i} - R_{B,i}, \quad i = 2, \cdots, N-1 \qquad (4b)$$

$$\frac{dN_N}{dt} + \frac{1}{V_T}\frac{dV_T}{dt}N_N + \frac{Q_sN_N - Q_eN_{Ne}}{V_T} - \frac{G(R_{N-1})}{2\Delta C_N}N_N - \frac{G(R_{N-1})}{2\Delta C_{N-1}}N_{N-1} = R_{A,N} - R_{B,N} \qquad (4c)$$

This set of equations can now be solved simultaneously for the two-phase bubble size distribution provided the growth rate, which is a function of the temperature and thus the heat transfer of the system, can be expressed adequately. We begin with the expression (Byrd and Mulligan, 1986) for the growth rate of a two-phase bubble as shown in Figure 1

$$G(R) = \frac{dR}{dt} = \frac{h\Delta T_r(\rho_L - \rho_v)}{\rho_L\,\rho_v\,h_{fg}} \qquad (5)$$

and then express the bubble heat transfer coefficient, h, as a function of the relative bubble velocity, radius, and physical characteristics,

$$h = h(U, R, \text{properties}) \qquad (6)$$

Thus, with the specification of Eq. 6, the bubble size distribution can be determined from Eqs. 4. Once the size distribution is known, the overall heat transfer Q can then be calculated as the sum of the heat transfer of the bubble classes. That is

$$Q = \sum_{i=1}^{N} Q_i = 4\pi \sum_{i=1}^{N} R_i^2 N_i h \Delta T_r \qquad (7)$$

The volumetric heat transfer coefficient, U_v is then found using

$$U_v = \frac{Q}{V_T(T - T_{\text{sat}})} \qquad (8)$$

This parameter is important in preliminary system design (Jacobs, 1988) as well as in predicting the overall heat transfer in an existing direct-contact heat exchanger.

The following noniterative computational procedure is used. The initial temperature of the dispersed and continuous liquids and the initial droplet size distribution are specified, along with the properties of the dispersed phases and the continuous phase. The volume of suspension, heat transfer coefficient, and total heat transfer are then calculated. From the total heat transfer, a new average continuous fluid temperature is calculated from the first law of thermodynamics, as

$$T(t + \Delta t) = \frac{Q_eT_i + \dfrac{V_T(t)T(t)}{\Delta t} - \dfrac{Q}{c_p\rho_c}}{\dfrac{V_T(t + \Delta t)}{\Delta t} + Q_s} \qquad (9)$$

This temperature is then used to find fluid properties at the next small time step, $t + \Delta t$. During the next time interval, evaporation of the injected droplets causes an increase in the suspension volume, V_T, calculated by

$$V_T(t + \Delta t) = \frac{V}{1 - \dfrac{4}{3}\pi \displaystyle\sum_{i=1}^{N} S_i^3 N_i} \qquad (10)$$

with the rate of change in volume as

$$\frac{dV_T}{dt}(t + \Delta t) = \frac{V_T(\mathrm{t}) - V_T(t - \Delta t)}{\Delta t} \qquad (11)$$

Also, new heat transfer characteristics and bubble growth are determined. Thus, a new size distribution is found using Eqs. 4.

This numerical solution is stable due to the fourth order Runga-Kutta subroutine used. To insure a valid, unique solution, however, the time increment is chosen such that the growth function is not so large as to displace a bubble through more than one class size per time step. Ten classes were chosen for this case. Examples involving a larger number of classes with smaller class sizes required smaller time increments to insure a unique solution and thus more computer time with little improvement in results. It must also be noted that in choosing smaller class sizes, correlation with experimental results would not be improved due to the small size differential of the bubble classes. In using experimental initial droplet sizes (R_0 = 1 mm to 3 mm) and reasonable time increments (Δt = 0.1 to 0.5 s), no problems with stability or uniqueness were found. Several values of Δt for each R_0 were chosen and compared with no difference in results.

Application

To illustrate the nature of the solution method in this work, an application was chosen for study which involves determining the transient cool-down of a batch crystallizer (or storage tank), as shown in Figure 2. Liquid refrigerant droplets (butane, pentane, etc.) of a uniform and known diameter are injected at a given rate at their saturation temperature into a tank which contains unstirred immiscible liquid water at a higher temperature. It is assumed that the surface tension of the drops is greater than that of the surrounding continuous liquid, a case common in applications. As time progresses, the droplets evaporate, cooling the water, with the vapor exiting the system through the top of the tank. The continuous liquid (water) is assumed not to circulate externally. Although this system may not be well-mixed geometrically, the overall population balance for V_T will be applicable as outlined by the above method. The two-phase bubble size distribution, overall heat transfer, volumetric heat transfer coefficient, and the transient average liquid water temperature

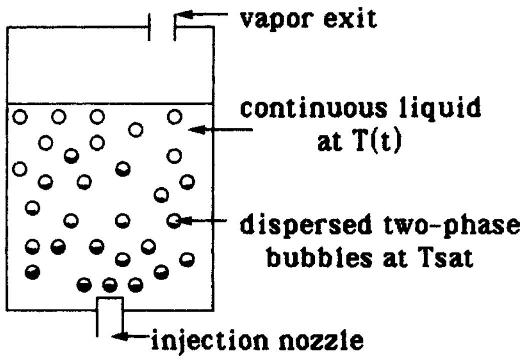

Figure 2. Evaporative spray column storage tank.

are to be determined. Although the proposed solution method has more general applications, this example was chosen due to its transient nature and the availability of necessary single-bubble heat transfer correlations.

Although no expression exists for the local heat transfer in a dispersion of bubbles, recent studies (Coban and Boehm, 1986) have shown that it can be assumed that when the number density of bubbles (void fraction) is sufficiently small, single-bubble heat transfer correlations yield reasonable results. Thus, a single-bubble relationship (Tochitani et al., 1977) is used, where

$$h = 0.2916 \frac{k_c}{R} \left(\frac{RU}{\alpha_c} \right)^{1/3} \left(\pi - \beta + \frac{\sin 2\beta}{2} \right)^{2/3} \qquad (12)$$

and

$$(3 \cos \beta - \cos^3 \beta + 2) = \frac{4(1 - x)}{1 + x(m - 1)} \qquad (13)$$

as illustrated in Figure 1, where $\beta = 0$ indicates a liquid droplet and $\beta = \pi$ is a fully evaporated vapor bubble. The bubble rise velocity due to evaporation for a single bubble, including change of shape and internal sloshing, is given by (Raina et al., 1984)

$$U_B = 1.91 \left[\left(1 - \frac{\rho_d}{\rho_c} \left(\frac{R_0}{R} \right)^3 \right) \left(\frac{\sigma_c}{\rho_c 2R} \right) \right]^{1/2} \qquad (14)$$

However, since the hydrodynamic effects surrounding each bubble are not always negligible and bubble/bubble interactions can affect the motion of individual bubbles, Eq. 14 is corrected (Gal-Or and Walatka, 1967) to yield

$$U = U_B \frac{(1 - \epsilon)^2}{1 - \epsilon^{5/3}} \qquad (15)$$

Computations were carried out with the water at an initial temperature of 5°C and a volume of 100 L. One hundred refrigerant droplets (butane) of radius 1 mm and temperature 0°C were assumed injected per second into the heat exchanger (with an initial distribution of 100 droplets). These conditions were not chosen arbitrarily for illustration but rather represent roughly the conditions of a crystallizer which was to be tested in our laboratory. Due to low bubble density and gentle flow conditions, random coalescence and breakage were assumed negligible. However, these factors can be important and represent an area of work for which very little information is available. Modeling these mechanisms was beyond the scope of the present study.

Results and Conclusions

The individual bubble heat transfer coefficient, as shown in Figure 3, varies with radius as empirically predicted by Raina and Grover (1988), decreasing as the bubbles grow. This behavior occurs because most of the heat transfer takes place at the interface between the continuous liquid and the liquid phase of the bubble, an area which rapidly decreases initially as the evaporation process begins. As the bubble grows, its heat transfer coefficient becomes less sensitive to growth, almost vanishing at approximately three times its original size. This is because the contact area remains relatively constant for $R/R_0 > 2$, as most of the bubble volume is occupied by vapor which has little effect on the heat transfer to the liquid. The angle β is also plotted for each R/R_0. As β approaches π asymptotically, it can be seen that h approaches zero and reaches a value close to zero at $R/R_0 = 3.5$, at which point it is assumed that the bubbles are totally evaporated and leave the system.

Population density as a function of R/R_0 is shown to vary over time in Figure 4. N represents the number of bubbles in each class with R/R_0 being the average radius of each full class. Thus, $R/R_0 = 1$ represents radius ratios from 1.0 to 1.25, $R/R_0 = 1.5$ represents ratios from 1.25 to 1.75, and so forth. Initially, it was assumed that the vessel contained 100 unevaporated droplets. Hence, the curve corresponding to $t = 0$ seconds indicates that all 100 droplets are of size $R/R_0 = 1$. For times greater than zero, it was assumed that 100 droplets/second are injected continuously. Thus, the number of bubbles in the system increases by the inlet flow, and the size range shifts to larger values of R/R_0 because of growth. This behavior is seen in the curve corresponding to $t = 4$ seconds for a total of 500 bubbles. At this time, the distribution reflects the number of droplets initially present at $t = 0$, as well as those injected over the first four seconds.

After 12 seconds, 1,300 bubbles are present, and the curve resembles a beta function, which has been used to characterize this relationship (Byrd and Mulligan, 1986) when moment methods of analysis are employed. After 12 seconds, the bubble

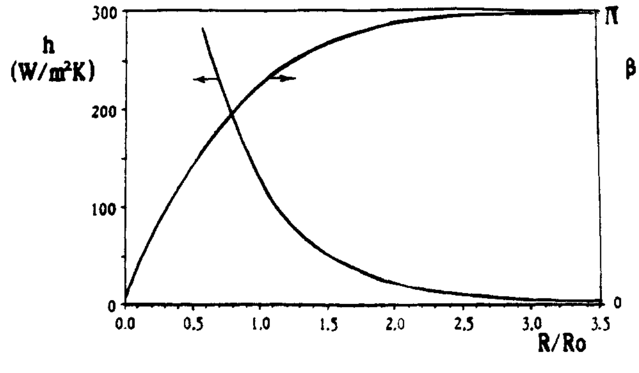

Figure 3. Bubble heat transfer coefficient and interface angle vs. dimensionless radius.

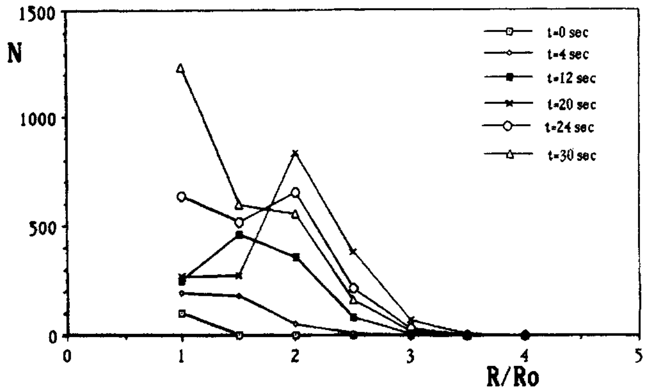

Figure 4. Bubble number density vs. dimensionless radius over time.

growth causes bubbles to fully evaporate ($\beta = \pi$) and thus exit the tank. This time for evaporation is greater than for steady-state applications due to a decrease in growth as the temperature difference between the bubbles and the continuum decreases.

At time $t = 20$ seconds, further growth is evident, as most of the bubbles are of $R/R_0 > 2$ and the curve is shifted to the right. At this time, 200 bubbles have fully evaporated and left the system. By 24 seconds, 500 bubbles have left the system. At $t = 30$ seconds, fewer bubbles are evaporating due to the decrease in growth near thermal equilibrium. As the temperature approaches steady state (i.e., no further heat transfer or bubble growth), the curve shifts toward the left. In the limit as the continuous liquid temperature approaches the saturation temperature of the bubbles, no further growth will be seen and all of the injected droplets will remain liquid. Hence, the limit on the rate of increase of droplets in the system is the rate at which the droplets are injected.

The volumetric heat transfer coefficient is shown to vary with the holdup fraction of the dispersed fluid in Figure 5 and to be parametrically dependent on the entering droplet size. Note from Figure 5 that for two-phase evaporating bubbles, total holdup includes both liquid and vapor fractions, and the liquid holdup includes only the liquid fraction of the dispersed fluid. The latter may be more pertinent in applications where liquid carryover is important, whereas the former might be more important when foaming is of interest. The solid line represents the instantaneous correlation of U_v and the total holdup fraction (liquid plus vapor) for the case of $R_0 = 1$ mm. Designated lines represent U_v vs. the holdup fraction of only the liquid phase (total holdup minus vapor fraction), computed at various instants of time and for several values of R_0. A dashed line is used to represent an empirical correlation (Plass et al., 1979) of U_v vs. the holdup fraction for direct-contact liquid-liquid heat exchangers.

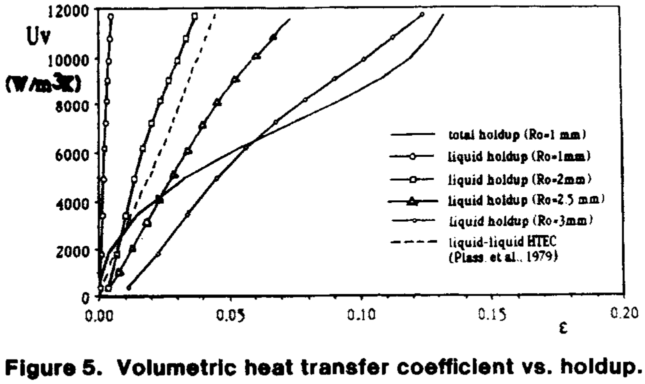

Figure 5. Volumetric heat transfer coefficient vs. holdup.

This correlation is for one continuous liquid and a monodispersed fluid of spherical droplets in the size range of 2 to 2.5 mm in radius and is included here for comparison since no data for two-phase bubbles could be found in the literature. The solid line shows very clearly the large expansion of the two-phase bubbles from the volume of an initially all liquid droplet and that this expansion does not occur with a commensurate increase in U_v. The curve is strongly shifted to higher total holdup, representing the large increase in vapor fraction, while the U_v increases only moderately. When U_v is plotted against liquid holdup (designated lines), it shows a strong dependence on inlet drop size R_0, and an interestingly close agreement with the empirical curve for liquid-liquid heat exchangers. The case of $R_0 = 2$ mm is within the experimental error of the empirical curve, which corresponds to particle sizes of approximately 2 mm to 2.5 mm in radius. A calculation at 2.5 mm is in excellent agreement and bounds the curve nicely. We believe this is due to the fact that the plate-like geometry of the liquid phase in each bubble (Raina and Grover, 1988) has about the same contact area with the continuous phase that the same volume of liquid in a spherical droplet would have. Hence, the empirical curve represents some degree of corroboration of the computed results.

The predicted total heat transfer rate changes with time (Figure 6), increasing as more droplets are injected and then decreasing as the effect of a temperature decrease outweighs the effect of an increase in the number of droplets (Figure 7). The heat transfer rate approaches zero as the temperature approaches the saturation temperature of the bubbles and the system reaches thermal equilibrium (after about 30 seconds). For comparison, the total heat transfer is calculated using the present model with experimental single-bubble heat transfer data (Sideman and Taitel, 1964), which was taken for single bubbles of different sizes and temperature differentials. This calculated curve is also shown Figure 6. The two curves are remarkably similar and in relatively good agreement. The formulated experimental curve lies above the predicted values of total heat transfer rate (which includes bubble interactions), which may be due in part to the lack of bubble-bubble

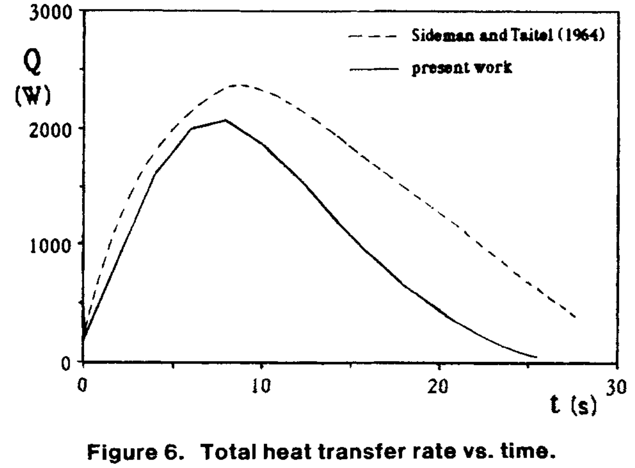

Figure 6. Total heat transfer rate vs. time.

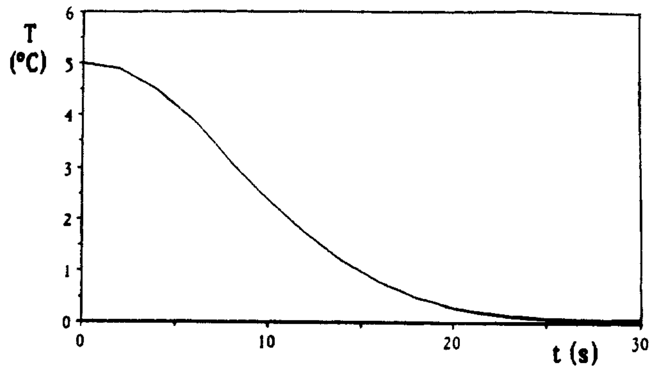

Figure 7. Continuous liquid temperature vs. time.

interaction effects in the formulated curve. As predicted by Gal-Or and Walatka (1967), bubble interactions cause a decrease in the bubble velocity (about 10% at total holdup $\epsilon = 0.05$ and about 23% at total holdup $\epsilon = 0.1$), causing a decrease in the heat transfer, which becomes more noted as the void fraction increases.

Thus, the method of classes is a useful tool in predicting the behavior of direct-contact evaporation systems. It has advantages over the method of moments in that general or empirical equations for the local heat transfer coefficient and bubble growth can be used and that no functional form of the bubble size distribution is required. Thus, a wide variety of cases can be studied with the method of classes, including the characteristics of steady-state heat exchanger systems. To fully utilize the method of classes in such cases, however, a more complete and accurate understanding of the heat transfer mechanisms in dispersions in complex flows is needed.

Notation

C = class of bubbles, m
c_p = specific heat of continuous fluid, J/kg · K
$G(R, t)$ = bubble growth rate, m/s
h = single-bubble heat transfer coefficient, W/m^2 · K
h_{fg} = latent heat of vaporization of dispersed phase, J/kg
k = thermal conductivity, W/m · K
m = ratio $\rho L/\rho v$
N_i = number of bubbles in class i per total volume
Q = total heat transfer rate, W
Q_e = volumetric flow rate of inlet, m^3/s
R = bubble radius, m
R_0 = initial droplet radius
S = characteristic size of bubble, m
t = time, s
T = continuous fluid temperature, °C
T_{sat} = saturation temperature of dispersed phase, °C
U = corrected single-bubble rise velocity, m/s
U_b = single bubble rise velocity, m/s
U_v = volumetric heat transfer coefficient, W/m^3 · K
V = continuous fluid volume, m^3
V_T = total suspension volume, m^3
x = weight fraction vapor

Greek letters

α = thermal diffusivity, m^2/s
β = half opening angle of vapor phase in bubble, rad
$\Delta T_r = T - T_{sat}$
ϵ = dispersed-phase holdup fraction
ϕ_i = constant volumetric density Ψ_i, 1/m^4
ρ = density, kg/m^3
σ = interfacial tension, N/m
$\Psi(R,t)$ = volumetric density function, 1/m^4

Subscripts

B = bubble
c = continuous phase
d = distributed phase
e = inlet
i = class index
L = liquid phase
N = final size
o = initial size
s = outlet
v = vapor phase

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